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Optics of Ion Beams of Arbitrary Perveance Extracted from a Plasma*

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A new iteration scheme is proposed for the solution of the ion optics of high current ion beams extracted from a plasma. The scheme (a) requires far less computational effort than other schemes; (b) converges for arbitrary perveance; (c) allows the solution of the problem far back into the extraction plasma and (d) is not geometry dependent, so it should be usable for a wide variety of situations.

1. INTRODUCTION

We consider the extraction of high current ion beams from a plasma and the subsequent acceleration of these beams by electrostatic fields. This problem is of interest in the design of ion thruster rocket engines and high power neutral beam injectors for heating plasma containment devices. Specifically, we consider plasma near a plane boundary with holes in it as is shown in Fig. 1. Subsequent to passing through a hole, ions from the plasma are accelerated by an applied electric field. The problem can conveniently be divided into three parts: (a) solution of Poisson's equation; (b) computation of ion trajectories with disposition of ion space charge; (c) iteration between (a) and (b) while inserting equilibrium electron space charge.

Two methods, which differ in the initial condition of the ions and the treatment of the plasma electrons, are currently employed to deal with this problem. The first method [1] starts the ions on a surface in the hole where the electric field is taken to be zero and through which electrons do not penetrate. This is an approximation of dubious validity since there is no region of space where both the electric field and the plasma electrons are inconsequential. The second method [2, 3] starts the ions on an equipotential surface in the sheath region of the plasma and includes space charge of electrons from the plasma which penetrates this surface. The approximate position, potential, and field of this surface as well as the initial directed ion speed are given approximately by a solution to the collisionless one-dimensional sheath problem [4]. A procedure has been worked out [2] which, given the emitting surface potential, adjusts the emitter position automatically so that the electric field is consistent with

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the 1–D solution. However, this presumes that the electric field is constant along such a potential surface in the 2–D problem. An alternative procedure is to place the emitting surface position farther into the plasma (~100 debye lengths from the boundary instead of ~10 debye lengths) where equipotential surfaces are essentially planer with both the position and potential specified from the 1–D solution [4]. The advantage of this procedure is that the emitter position does not need to be varied

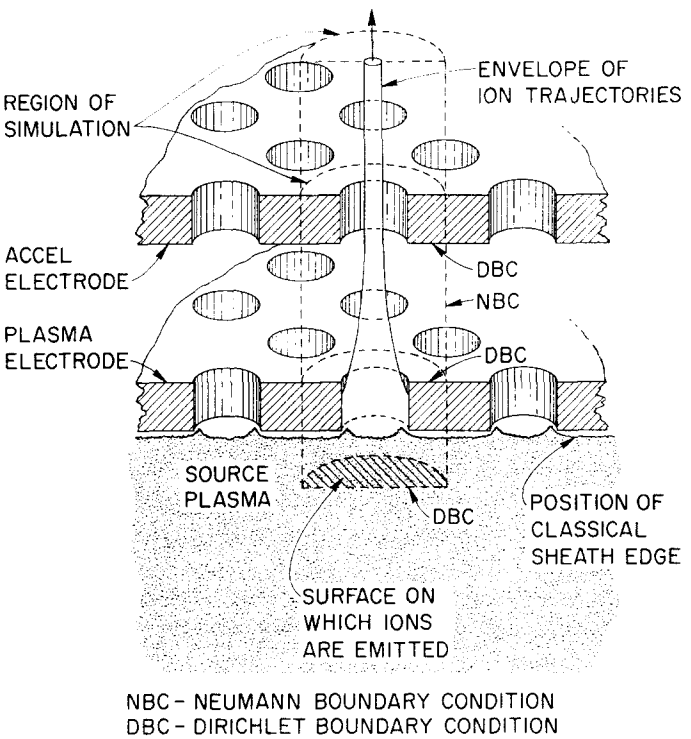

FIG. 1. Illustrative arrangement of high current ion source showing region of numerical simulation and boundary conditions on both potential and ion distribution function.

throughout the calculation and the two-dimensional sheath problem is solved directly. The disadvantage is that Poisson's equation must be solved in a larger region in which the source terms due to the ions and electrons are large and almost cancelling.

A principal difficulty with the method in Ref. [2, 3] is that, with the inclusion of the plasma electrons, convergence of this very nonlinear problem is not trivial. In Section 2 we will review briefly the structure of the calculation we use. In Section 3 we examine a previously used iteration scheme [3] and propose two different iteration schemes, both of which may involve orders of magnitude less computing and one of which can be used for arbitrary perveance. These two schemes we call the simultaneous convergence scheme and the accelerated under-relaxation averaging scheme. In Section 4 we consider an example illustrating the iteration scheme considered.

2.A Formulation

We consider the simultaneous solution of: (1) Poisson's equation for the electric potential ϕ,

$$\nabla^2\phi = \frac{e}{\epsilon_0}(N - n), \tag{1}$$

where ϵ_0 is the permittivity of free space, e is the magnitude of the electronic charge, and n is the ion density,

$$n = \int f\, dv, \tag{2}$$

v and f are the ion velocity and distribution function respectively; (2) Vlasov's equation, which in the absence of magnetic fields is

$$v \cdot \nabla f - \frac{e}{m}\nabla\phi \cdot \nabla_v f = 0, \tag{3}$$

with a Boltzmann distribution [2] of electron densities,

$$N = N_0 \exp(e\phi/kT) \tag{4}$$

where ϕ is zero in the plasma and negative elsewhere, N_0 and T are the electron density and temperature in the plasma, and m is the ion mass.

The solution of Eqs. (1)–(4) is sought in a region bounded by a surface where ϕ, or $\nabla\phi$, and f, or ∇f, are specified. See Fig. 1.

To take into account, in some sense, ion collisions with neutrals in the plasma, the ions are assigned a transverse initial velocity parallel to the sheath surface. One can either presume a Maxwellian ion distribution or use the ion velocity distributions obtained in the one-dimensional collisional sheath problem [5]. The initial ion transverse velocity causes a finite angular momentum which needs to be considered in 2–D problems phrased in cylindrical coordinates.

2.B Solution

Poisson's equation, Eq. (1), is solved by a finite difference method on a mesh with given source terms, using the method of successive over-relaxation [6]. Boundaries are treated with a procedure due to Hornsby [7]. The over-relaxation coefficient is automatically chosen using a method due to Carré [8] for the maximum rate of convergence. The rectangular mesh is set up automatically by the program [7]. A stretched grid in the axial direction is chosen such that mesh points near the emitter are relatively close together.

Vlasov's equation, Eq. (3), is solved indirectly by solution of orbit equations for many initial orbits entering from the emitting surface toward the hole with velocities

appropriate to the initial velocity distribution specified. Considering electric fields produced by the solution of the Poisson equation and centripetal forces, the orbit equations are solved by a deferred limit integrator [9] and ion charge is deposited on the mesh points in accordance with the continuity equation.

Electron charge is deposited directly on the mesh points via Eq. (4).

3. Iteration Schemes

(a) The Sequential Convergence Scheme. This scheme, which has been used previously [3] converges everything separately as is shown in Fig. 2. Initially there are no source terms and Laplace's equation is iterated [6, 7]. The iteration continues until the relative change in potential at every mesh point is smaller than an error parameter (typically 10^{-4}). From these vacuum fields electron space charge is deposited from Eq. (4). Then ion orbits are calculated and space charge deposited. At this point Poisson's equation is iterated in an SOR scheme [6] with no addendum to the source

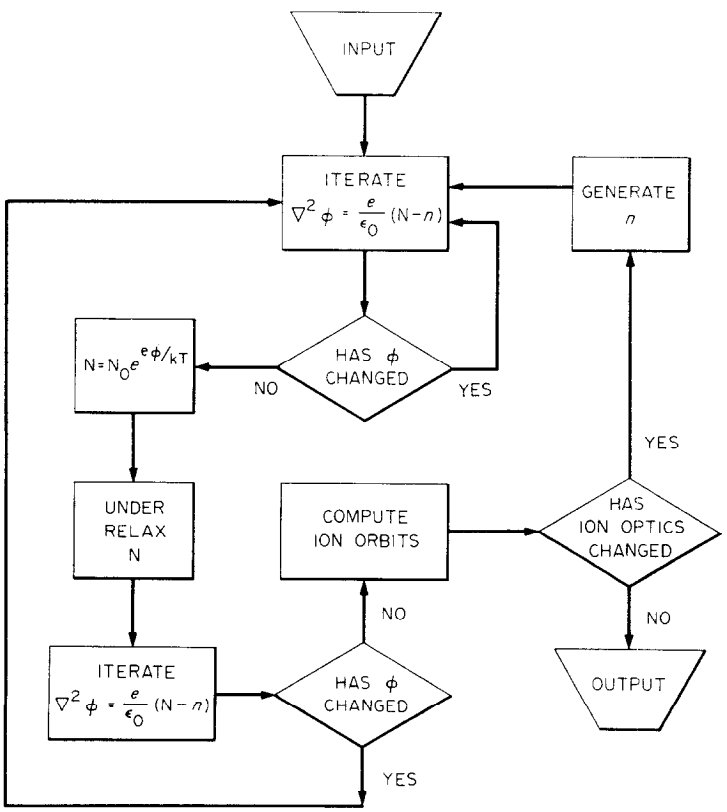

FIG. 2. Flow diagram for sequential convergence scheme.

terms (hereafter called linear iterations) until the potential is relatively unchanged from the previous iteration. The electron space charge is then recalculated from the resulting potential and under-relaxed, i.e.,

$$N_{\text{new}} = \alpha N_{\text{new}} + (1 - \alpha) N_{\text{old}} , \tag{5}$$

where α is the electron under-relaxation coefficient (typically 10^{-2}), which in this scheme is held fixed. After the electron charge is under-relaxad via Eq. (5), Poisson's equation is iterated again. If after the first iteration the potential is changed, then the Poisson equation with fixed source terms is iterated again until converged as described above. If after the first iteration the potential remains unchanged, then the ion orbits are computed and ion space charge deposited. Eventually, when the ion orbits are computed and ion space charge is deposited on the mesh points, the whole process of linear iteration convergence and electron iteration convergence is started over until the ion optics, or the resultant potential, converges.

This scheme uses the rationale that it is easier to compute the electron space charge

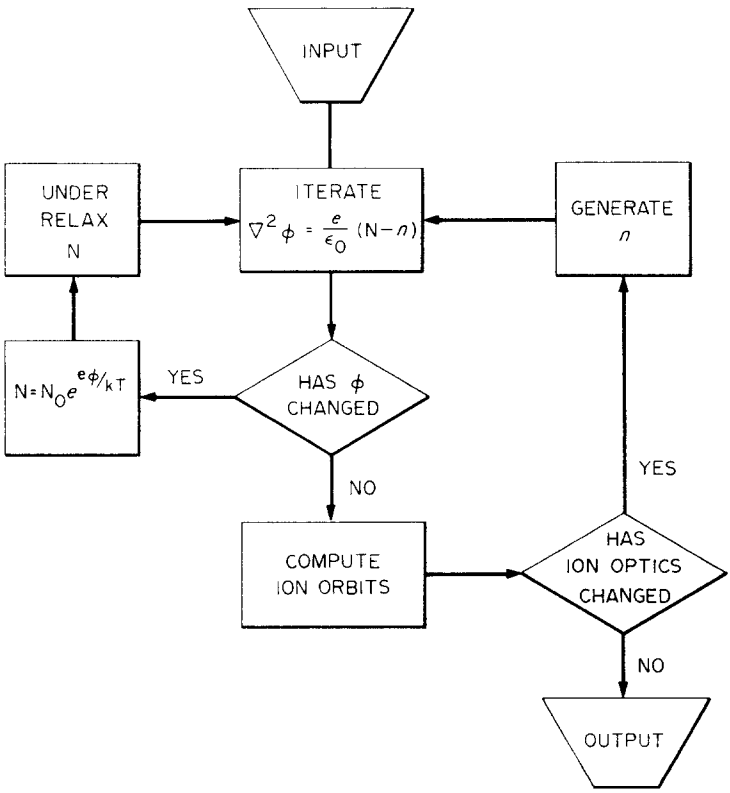

FIG. 3. Flow diagram for simultaneous convergence scheme.

via Eq. (4), than the ion space charge by Eqs. (2) and (3) and, therefore, the electrons are iterated more often than the ions.

(b) The Simultaneous Convergence Scheme. This scheme, which is shown in Fig. 3, is very similar to scheme (a) except that it short-cuts the requirement of linear convergence before electron space charge is inserted. Each time Poisson's equation is iterated and the potential changes from the previous iteration, the electron term is updated. Therefore, during each iteration both the linear and electron iterations are converging simultaneously.

(c) The Accelerated Under-relaxation Averaging Scheme. This scheme is a significant departure from schemes (a) and (b), as can be seen from Fig. 4: (1) no

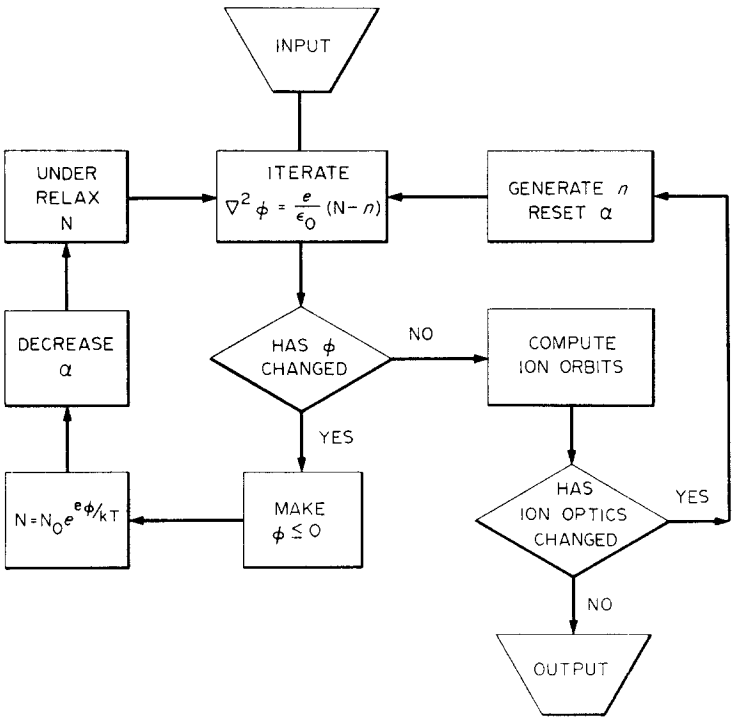

FIG. 4. Flow diagram for accelerated under-relaxation averaging scheme.

attempt is made at converging the electron and/or linear iterations; (2) the potential is modified if necessary at every step of the calculation; (3) the electron under-relaxation is varied throughout the calculation. The modifications are made for dealing with arbitrary perveance problems. In high perveance cases the space charge forces in the extraction region may be extremely large and almost cancelling. Because of the extreme nonlinearity of the problem, we choose to do a very smoothly damped

average. If the iteration scheme presents us with a potential at some point that is more extreme than the extraction plasma potential, we set that potential to the plasma potential (or $\approx$ the emitter surface potential). We are justified in doing this since from the 1–D analysis [4] the potential can never be more extreme than the plasma potential. In this scheme electrons are not under-relaxed (i.e., $\alpha = 1$) the first time they are inserted after each ion iteration. Thereafter α is divided by a number, D, slightly bigger than unity (typically 1.02) upon each electron insertion. This effects an average over oscillations that tend to occur. The scheme iterates the ions when the potential changes little between iterations. This is bound to happen for any case, since eventually α will become so small that no change is being made in anything during the iteration. This procedure apparently converges, solves Eq. (1) in all the cases examined, and agrees with schemes (a) and (b) where their results are available.

4. Example

For a typical ion source as shown in Fig. 1, we compare in Fig. 5 the number of linear iterations required for convergence as a function of relative perveance. Relative perveance is defined as the perveance divided by the Child-Langmuir [10] space charge limited perveance obtained for ions without electrons, in the 1–D diode, whose length is the same as the gap between the accelerating electrodes and whose applied potential is the same as that considered in the cylindrical case. In Fig. 5 the most convergent ion optics, and therefore the region of perveance of greatest interest, is indicated on the axis. The emitting surface is taken 100 debye lengths into the source plasma from the lower electrode. This is considerably farther back into the plasma than the classical sheath edge which for the 1–D problem is typically about 15 debye lengths.

Shown in Fig. 6 is a typical solution for the ion trajectories and potentials (bottom) and the potentials calculated initially considering only the boundary conditions neglecting ion or electron space charge (top) with ion trajectories. The left most potential contour in Fig. 6 is a potential halfway between the source plasma potential and the potential on the first electrode. In the converged solution (bottom of Fig. 6) this potential contour is near the sheath edge. The emitting surface is 100 Debye length to the left of the first electrode and the relative perveance, P/P_{CL}, is 1/3.

In Fig. 5 we see that scheme (a), the sequential convergence scheme, requires the most computational effort and converges for the smallest range of perveance. Scheme (b), the simultaneous convergence scheme, requires far less computational effort and has a greater range of perveance in which convergence is possible. By making α smaller, as shown in Fig. 5, the range of perveance for which convergence is possible increases somewhat but the effort at smaller perveance is greater. Scheme (c) the accelerated under-relaxation averaging scheme appears to converge for all perveance considered in Fig. 5. The accelerating parameter D can be made bigger allowing even fewer iterations than shown in Fig. 5 but with a gradual loss in accuracy.

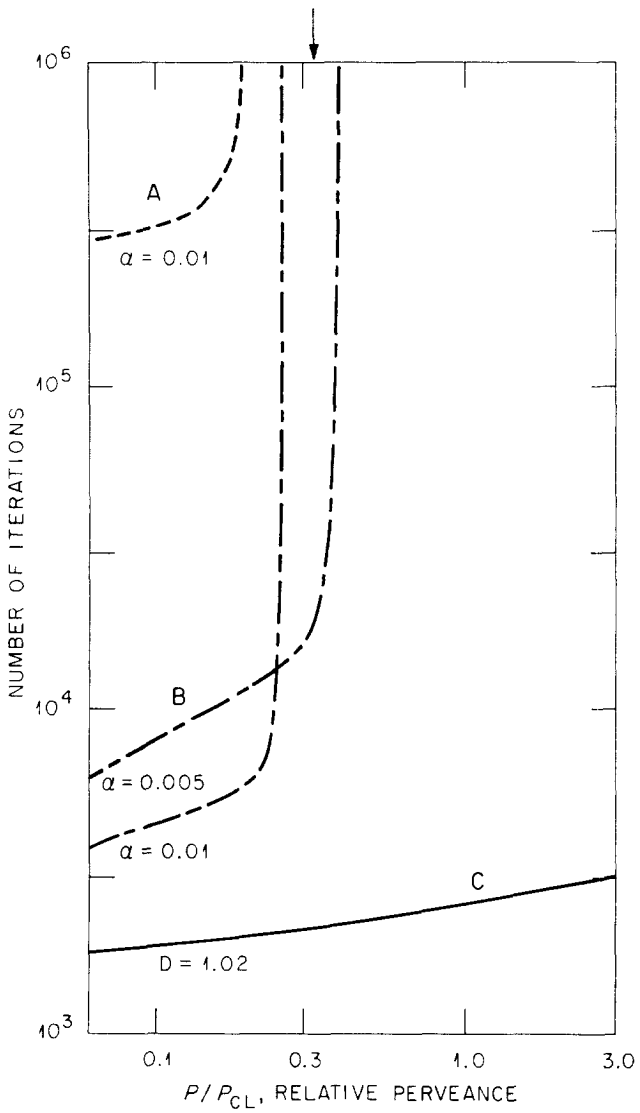

FIG. 5. Comparison of computational time for the three schemes considered as a function of perveance on the scale of the Child-Langmuir perveance. The arrow on the abscissa is the perveance where the ion beam is most convergent.

Schemes (a) and (b) can be made to converge at higher relative perveance than shown in Fig. 5 if the emitter position is moved closer to the electrode. However, as mentioned before, in so doing the stipulation of the emitter conditions becomes less reliable.

Since scheme (c) is not geometry-dependent, it should be usable for slot geometry or other configurations. Since it is explicit it can be used directly with a finite element as well as a finite difference computation.

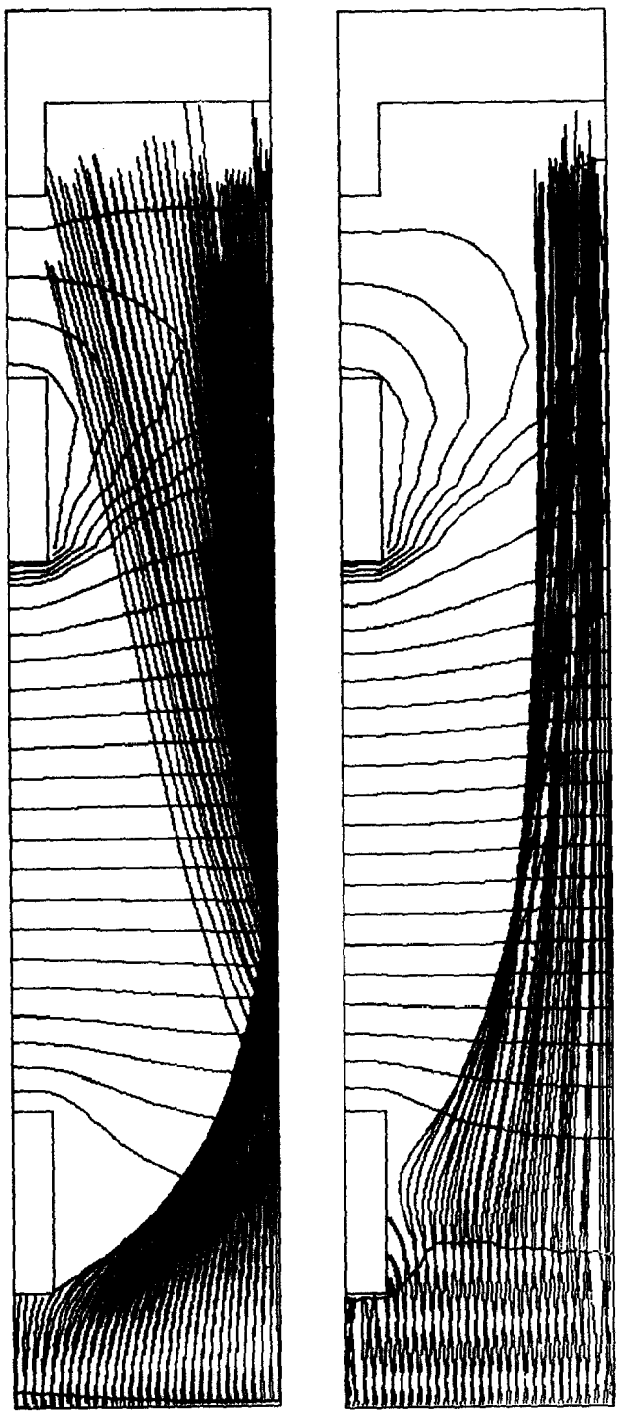

FIG. 6. Typical converged solution (bottom) and initial iteration in vacuum fields (top) for the case considered in Fig. 5 at a relative perveance of 1/3.

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